'समानो मन्त्रः समितिः समानी'

UNIVERSITY OF NORTH BENGAL

B.Sc. Programme 2nd Semester Examination, 2023

DSC1/2/3-P2-STATISTICS

PROBABILITY THEORY AND DISTRIBUTIONS

Time Allotted: 2 Hours   Full Marks: 40

The figures in the margin indicate full marks.

GROUP-A

1. Answer any ***five*** questions: 1×5 = 5

(a) If events A and B are not mutually exclusive, then show that $P(AB) \geq P(A) + P(B) - 1$

(b) State two properties of Poisson distribution.

(c) For a binomial distribution with mean 5 and S.D 2, find the mode.

(d) What is the chance that a leap year selected at random will contain 53 Sundays?

(e) Show that variance of the standard normal variable is 1.

(f) If the events A and B are independent, show that A^c and B^c are also independent.

(g) Distinguish between p.m.f and p.d.f.

(h) If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$ and $P(A^c) = \frac{1}{2}$, then show that A and B are independent.

GROUP-B

2. Answer any ***three*** questions: 5×3 = 15

(a) State and prove Bayes' Theorem.

(b) Derive Poisson distribution as the limit of binomial distribution.

(c) Find the points of inflection of the normal curve.

(d) Prove that the variance of binomial distribution is npq.

(e) If X is a Poisson variate such that $P(X = 2) = 9P(X = 4) + 90P(X = 6)$. Find the mean of X.

GROUP-C

3. Answer any ***two*** questions: 10×2 = 20

(a) (i) A coin is tossed until a head appears. What is the expectation of the number of tosses?

(ii) Show that odd order central moments of the normal distribution are equal to zero.

(b) (i) Show that the expectation of the product of two independent random variables is equal to the product of their expectations.

(ii) Show that for the binomial distribution $\mu_{r+1} = p(1-p)\left(nr\mu_{r-1} + \frac{d\mu_r}{dp} \right)$

where the symbols have their usual meanings.

(c) (i) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses if experience show that 2% of such fuses are defective.

(ii) The joint p.d.f. of (X, Y) is given by

$f(x, y) = 2$; $0 < x < 1$, $0 < y < x$

$= 0$; otherwise

Find the marginal density of X and the conditional density of Y given $X = x$.

(d) (i) If X follows binomial distribution with parameter n and p then prove that $P(X \text{ is even}) = \frac{1}{2}[1 + (q-p)^n]$, where $p + q = 1$

(ii) Let the variable X have the distribution $P(X = 0) = P(X = 2) = p$, $P(X = 1) = 1 - 2p$, for $0 \le p \le \frac{1}{2}$. For what value of p is the $\text{var}(X)$ maximum?

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